Extensions of semigroups by symmetric inverse semigroups of a bounded finite rank

Oleg Gutik
(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)
E-mail: `oleg.gutik@lnu.edu.ua, ovgutik@yahoo.com`

Oleksandra Sobol
(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)
E-mail: `o.yu.sobol@gmail.com`

We study the semigroup extension $\mathscr{I}_\lambda^n(S)$ of a semigroup S by symmetric inverse semigroups of a bounded finite rank.

Definition 1. An subset D of a semigroup S is said to be ω-unstable if D is infinite and $aB \cup Ba \nsubseteq D$ for any $a \in D$ and any infinite subset $B \subseteq D$.

Definition 2. An subset D of a semigroup S is said to be *strongly ω-unstable* if D is infinite and $aB \cup Bb \nsubseteq D$ for any $a, b \in D$ and any infinite subset $B \subseteq D$.

It is obvious that a subset D of a semigroup S is strongly ω-unstable then D is ω-unstable.

Definition 3. An *ideal series* (see, for example, [1]) for a semigroup S is a chain of ideals

$$I_0 \subseteq I_1 \subseteq I_2 \subseteq \cdots \subseteq I_n = S.$$

We call the ideal series *(strongly) tight* if I_0 is a finite set and $D_k = I_k \setminus I_{k-1}$ is an (strongly) ω-unstable subset for each $k = 1, \ldots, n$.

A finite direct product of semigroups with tight ideal series is a semigroup with a tight ideal series and a homomorphic image a semigroup with a tight ideal series with finite preimages is a semigroup with a tight ideal series too [2].

Proposition 4. *Let S be a semigroup which admits a strongly tight ideal series. Then the direct power $(S)^n$ admits a strongly tight ideal series too.*

Theorem 5. *Let λ be an infinite cardinal and n be a positive integer. If S is a finite semigroup then*

$$I_0 = \{0\} \subseteq I_1 = \mathscr{I}_\lambda^1(S) \subseteq I_2 = \mathscr{I}_\lambda^2(S) \subseteq \cdots \subseteq I_n = \mathscr{I}_\lambda^n(S)$$

is the strongly tight ideal series for the semigroup $\mathscr{I}_\lambda^n(S)$.

Theorem 6. *Let S be a semigroup which admits a strongly tight ideal series. Then for every non-zero cardinal λ and any positive integer $n \leqslant \lambda$ the semigroup $\mathscr{I}_\lambda^n(S)$ admits a strongly tight ideal series too.*

Definition 7 ([2])**.** An algebraic semigroup S is called *algebraically complete* in the class of semitopological semigroups $\mathfrak{S}$, if S with any Hausdorff topology τ such that $(S,\tau)\in\mathfrak{S}$ is H-closed in $\mathfrak{S}$.

Theorem 8. *Let S be an inverse semigroup which admits a strongly tight ideal series. Then for every non-zero cardinal λ and any positive integer $n \leqslant \lambda$ the semigroup $\mathscr{I}_\lambda^n(S)$ is algebraically complete in the class of Hausdorff semitopological inverse semigroups with continuous inversion.*

References

[1] A. H. Clifford and G. B. Preston. *The Algebraic Theory of Semigroups*, volume. I of *Amer. Math. Soc. Surveys 7*. Providence, R.I., 1961; volume. II of *Amer. Math. Soc. Surveys 7*. Providence, R.I., 1967.

[2] O. Gutik, J. Lawson, and D. Repovš. Semigroup closures of finite rank symmetric inverse semigroups. *Semigroup Forum*, 78(2): 326–336, 2009.